An Account of the Pendulum Observations connecting Kew and Greenwich Observatories made in 1903.

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Preliminary and Apparatus.

§ 1. The observations, of which a brief account is here given,* had their origin in the decision of the Government of India to resume the pendulum work which was brought to a close in 1870. Professor F. R. Helmert, Director of the Central Bureau of the International Geodetic Association, to whose advice the India Office is much indebted, recommended the use of a half-seconds pendulum equipment as designed by Colonel von Sterneck. This equipment was ordered through the Geodetic Institute at Potsdam, and the constants for the necessary pressure and temperature corrections were determined there by Professor L. Haasemann, under Professor Helmert's direction. A redetermination of these constants was made at Kew, at Professor Helmert's suggestion, and results were obtained in very close accordance with those found at Potsdam.

The apparatus gives only relative determinations of gravity; it was thus necessary to select a base station. As Kew Observatory had been the base station of the older Indian pendulum observations it was again selected, Dr. Glazebrook, Director of the National Physical Laboratory, having given permission and promised all necessary assistance. Meantime, a suggestion was made by the Astronomer Royal, and accepted by the Secretary of State for India, that the opportunity should be taken of swinging the pendulums also at Greenwich, thus allowing of a fresh intercomparison of g at Greenwich and Kew.

The apparatus was made by E. Schneider, of Vienna, after Colonel von Sterneck's design. There are four pendulums, Nos. 137, 138, 139, and 140, having times of vibration which are very nearly equal, and slightly in excess of half a second. The pendulums are of brass, heavily gilded, provided with agate edges on which to vibrate. Each carries a small vertical mirror fastened to its head, just above the line of the agate edges. The stand is also of brass, in the form of a truncated cone. It rests on foot screws, which

* The full account will be printed in the publications of the Great Trigonometrical Survey of India.

can be firmly clamped, and carries a highly polished agate plane for the reception of the agate edges of the pendulums.

The pendulum is started swinging with any desired amplitude by means of a lever in the base of the stand. The pendulum swings in air at atmospheric pressure, but is protected from draughts by a cover.

The most essential remaining part of the apparatus is a flash box, containing a contrivance whereby a shutter under the control of a break-circuit clock allows a flash of light to pass through a slit at every beat or alternate beat. This flash is reflected by the mirror on the half-seconds pendulum into a small telescope above the flash box. The times when the flash passes the horizontal wire in the telescope's field correspond to the coincidences of the half-seconds pendulum and the clock. The coincidence interval, c, is connected with the time of vibration, s, of the half-seconds pendulum by the equation

$$s = c/(2c-1).$$

In the present case we have approximately

$$c = 35 \text{ secs.} \quad \text{and} \quad s = 0{\cdot}507 \text{ sec.}$$

The amplitude of swing is observed by means of a scale on the front of the flash box, at a measured distance from the mirror on the half-seconds pendulum. The most convenient initial semi-arc is from 12′ to 15′.

The clock belonging to the apparatus, S.R. 238, was constructed by Messrs. Strasser and Rohde, of Glashütte; its pendulum, made by Riefler, of Munich, is of invar.

Use was also made of the sidereal standard clock at Greenwich, and at Kew of the clock Morrison 8702. Two break-circuit chronometers were also generously lent by Messrs. T. Mercer and Sons and Mr. V. Kullberg at a time when it was feared that S.R. 238 would not be ready for use. The Mercer chronometer was employed during two sets of observations at Kew.

The equipment included thermometers by Messrs. Negretti and Zambra. A barometer and hygrometer were lent by the National Physical Laboratory.

Corrections to Observed Time of Swing.

§ 2. Reduction to a vacuum was made by the formula

$$-\mathrm{K}'(\mathrm{B}+b)\left(1-\frac{3}{8}\frac{e}{\mathrm{B}}\right)\div(760+2{\cdot}79t),$$

where B is the external barometric pressure, e the pressure of aqueous vapour, b any excess of pressure inside the case—all in millimetres of mercury—t the temperature centigrade, and K′ a constant. The values of

K′ for the four pendulums were very nearly equal; the means obtained at Potsdam and Kew were respectively 595×10^{-7} and 605×10^{-7}.

The reduction to 0° C. was made by the formula $-Kt$, the mean values found for K at Potsdam and Kew being respectively $49{\cdot}0 \times 10^{-7}$ and $49{\cdot}4 \times 10^{-7}$. The Potsdam values of K′ and K were employed in the reductions, but the substitution of the Kew values would in no way have modified the conclusions as to the relative values of g at Greenwich and Kew.

The reduction to an infinitely small arc—in all cases a trifling correction —was made by the formula $-s\alpha^2/16$, where s denotes the time of vibration and α the mean semi-arc.

The results were also reduced to an ideal "rigid pillar" by a method used by Kater and developed recently by Professor R. Schumann, of the Prussian Geodetic Institute. Two pendulums vibrating in very nearly equal times are simultaneously suspended from the stand, their planes of vibration being the same. One is suddenly set swinging with considerable amplitude, the other being initially at rest. Observations of the rate at which the second takes up an oscillation from the first supply the means of calculating the small virtual increase in the length of the pendulum, which corresponds to the elastic flexure of the stand and pillar carrying it. The correction thus obtained to the time of swing—the stand being firmly clamped to a granite slab, cemented to a solid masonry pillar about 20 inches high—averaged about -38×10^{-7} second, the mean being nearly the same at Kew and Greenwich.

In the application of the temperature correction mentioned above it is tacitly assumed that the temperature recorded by the thermometer is that simultaneously possessed by the pendulum. In reality, when the change is at all rapid, the mean temperature of the pendulum lags behind that of the thermometer, and a further correction may thus be necessary, which depends on the rate of change of temperature. This "lag" correction was not determined directly for the apparatus in question, but was assumed to be 25×10^{-7} second for a rate of change of 1° C. per hour, that being the result obtained for an almost identical apparatus belonging to the Prussian Geodetic Institute. The mean values of the calculated lag corrections at Greenwich and Kew in the final comparisons were respectively $+0{\cdot}8 \times 10^{-7}$ and $+4{\cdot}4 \times 10^{-7}$ second, temperature always rising during the observations at Kew, and generally at Greenwich. The relative smallness of the correction at Greenwich was due to the temperature being easier to control, the room being larger than that at Kew, and fitted with electric light instead of gas.

Observations and Results.

§ 3. The observations were made as follows, the time of a coincidence being known very approximately in advance. Ten consecutive coincidences were observed; then, after an interval corresponding to 50 coincidences, ten more consecutive coincidences were observed. Ten values of a 60-coincidence interval were thus obtained and their mean taken. After 12 hours the observations were repeated, and a second mean obtained.

Under normal conditions the mean of these two means should be but little influenced by irregularities in the rate of the clock during the 24 hours. The observations were usually made from 9 P.M. to 1 A.M., and from 9 A.M. to 1 P.M. The rate of the clock or chronometer used was derived from star observations at Greenwich. When at Kew, use was made of the Greenwich 10 A.M. and 1 P.M. time signals, any necessary corrections to these being supplied by the Astronomer Royal.

At Kew the pendulums were swung—approximately in the plane of the prime vertical—in the North Room (formerly the platinum thermometer room) of the small house lying to the west of the main building. The position was 100 feet 6 inches west of, 5 inches north of, and 6 feet 4 inches higher than that occupied by Mr. G. R. Putnam in 1900, and by the Kater pendulums swung by Mr. E. G. Constable in 1888. Its coordinates are 51° 28′ 6″ N. Lat., and 0° 18′ 48″ W. Long.; its height above mean sea level 23 feet. At Greenwich the pendulums were swung in the plane of the meridian. The station was the same as that occupied by Colonel von Sterneck, Mr. Putnam, Mr. Hollis, and others. It is in the Record Room, about 20 yards east of the prime meridian. Its coordinates are 51° 28′ 38″ N. Lat, 0° 0′ 1″ E. Long.; its height above mean sea level 155 feet.

§ 4. The first observations were made by Major S. G. Burrard, Mr. E. G. Constable, and Major G. P. Lenox-Conyngham at Kew from June 22 to June 25, 1903, using only the Morrison clock, the Strasser and Rohde clock not having arrived from Germany. On the arrival of the latter clock, it was employed during observations, first at Greenwich, then at Kew, between June 29 and July 9. The results, however, proved discordant on reduction and were rejected, the clock at the makers' request being returned to them to be overhauled. Major Burrard having meantime had to embark for India, the final observations were made by Mr. Constable and Major Lenox-Conyngham. These consisted of observations at Kew from October 14 to October 16 and October 27 to October 31, with intermediate observations at Greenwich from October 20 to October 24. During each 24 hours two observations were made with each pendulum, employing the observatory clock,

and a further two with either the Strasser and Rohde clock or the Mercer chronometer.

Taking the mean of the corrected times of swing from the four pendulums the final result was:—

At Greenwich	0·5067036 sec.
Kew	0·5067001 ,,

The probable errors in these times were $\pm 3{\cdot}6 \times 10^{-7}$ second at Greenwich and $\pm 3{\cdot}1 \times 10^{-7}$ at Kew.

The mean difference in the times of swing was 35×10^{-7} seconds, and its probable error $\pm 4{\cdot}5 \times 10^{-7}$ second.

The results for the difference in the time of swing from the individual pendulums were as follows:—

Pendulum number	137	138	139	140
(Excess at Greenwich) $\times 10^7$	34	35	36	35

The agreement between the different pendulums is thus remarkably good.

§ 5. For g at Kew the value 981·200 cm. second^{-2} has been accepted by Professor Helmert.* This depends on the preliminary result of the determination of g at Potsdam, viz., 981·274.

If s_1 and s_0 be the times of swing at two stations where g_1 and g_0 are the values of g, then

$$s_1^2 g_1 = s_0^2 g_0,$$

or where $(s_1 - s_0)/s_0$ is small,

$$g_1 = g_0 \{1 - 2(s_1 - s_0)/s_0\}.$$

Hence, assuming the value 981·200 at Kew, and the above-mentioned excess in s at Greenwich, we have at Greenwich

$$g = 981{\cdot}186,$$

with a probable error of $\pm 0{\cdot}002$.

Comparison with Theory.

§ 6. Helmert's formula is

$$g = 978{\cdot}000\,(1 + 0{\cdot}005310 \sin^2 \phi) \left\{1 - \frac{2h}{\mathrm{R}} + \frac{3h}{2\mathrm{R}}\,\frac{\delta}{\Delta} - \frac{3h'}{2\mathrm{R}}\,\frac{(\delta - \theta)}{\Delta} + y\right\},$$

where ϕ is the latitude, h the height above sea level, h' the thickness of surface strata of low density, R the earth's mean radius, δ the assumed mean surface density (2·8), θ the actual density of surface strata, Δ the earth's mean density (5·6), y an orographical correction.

* "Bericht über die relativen Messungen der Schwerkraft mit Pendel-Apparaten," 'Report Geodetic Conference of 1900,' p. 321.

The y correction is inappreciable for such flat country as surrounds Kew and Greenwich.

A detailed examination, by Mr. A. Strahan, F.R.S., of the geological evidence as to the surface strata near the two observatories has led him to the following conclusions:—

At Kew there are 1140 feet of strata of average specific gravity 2·06—including 150 feet of London Clay—and 97 feet of limestone, or in all 1237 feet of strata above the palæozoic floor. At Greenwich the London Clay and the limestone are absent, and the estimated depth of the palæozoic floor is 933 feet. The calculated corrections to Helmert's standard value, 978·000 at latitude 0°, are as follows:—

Correction for	At Kew.	At Greenwich.	Difference in corrections. Kew – Greenwich.
Latitude	+3·178	+3·179	–0·001
(h/R) term	–0·002	–0·015	+0·013
δ/Δ "	+0·001	+0·006	–0·005
$(\delta-\theta)/\Delta$ term	–0·011	–0·009	–0·002
Total	+3·166	+3·161	+0·005

Thus the final *theoretical* values are—

Kew .... 981·166
Greenwich ... 981·161

The observed values are thus, in both cases, slightly in excess of the calculated, and the difference between them is greater than the calculated difference.

Historical Summary.

§ 7. The following table enumerates the differences between g at Kew and Greenwich as found in the present and in previous direct comparisons:—

Year.	Observers.	Method employed.	Kew less Greenwich.
1831 1873	Sabine .... Heaviside	Seconds pendulums	+0·069
1881	Herschel...............	Kater's pendulums, Nos. 4 and 6	+0·038
1888	Constable .... Hollis	Kater's pendulums, Nos. 4, 6, and 11...	+0·028
1900	Putnam	Three half-seconds pendulums	+0·012
1903	Burrard .... Constable .... Lenox-Conyngham...	Four half-seconds pendulums	+0·014

It is satisfactory to note that the results obtained with the modern forms of apparatus agree well with one another and do not differ greatly from the theoretical value. This encourages the hope that henceforth the pendulum may prove as satisfactory in practice as it has always been attractive in theory.

On the Behaviour of Certain Substances at their Critical Temperatures.

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A work entitled "Le Point Critique des Corps Purs" has recently been published by E. Mathias, whose opinion on matters relating to the critical state must always carry weight. In this he discusses at length the various theories which have been put forward to explain certain irregularities observed in the behaviour of substances, which were supposed to be pure, at their critical temperatures. These irregularities are not accounted for by the simpler theories of Andrews and Van der Waals. He calls attention to the experiments of certain investigators, which appear to suggest that the currently-accepted values of the critical constants of many common substances may be vitiated, either owing to the time allowed for the establishment of equilibrium between the coexisting phases near the critical point being insufficient, or the temperature at which the dividing surface vanishes not being independent of the relative masses of the two phases at the temperature at which this takes place.

According to I. Traube, substances contain different kinds of aggregates, which he calls "gasogenic" and "liquidogenic" molecules. It follows that if equilibrium demands that there shall be a certain concentration of these molecules in the vapour and liquid phases respectively, then unless dissociation and association take place instantaneously, there must elapse a time, following any change of condition, before equilibrium can be established between the two phases.

P. de Heen's theory goes further, and, assuming the existence of such complexes, suggests that it is possible that the concentration of them in the two phases is a function not only of the temperature, but also of the relative masses of the two phases, or, in other words, of the mean specific volume of the system under investigation. In this case a system